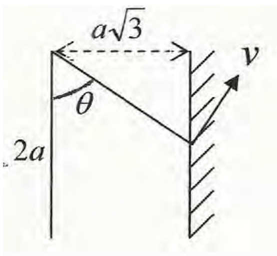
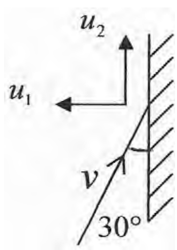
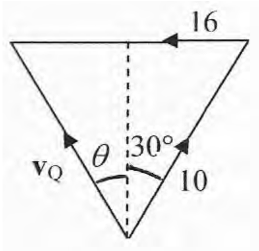
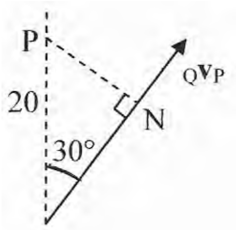
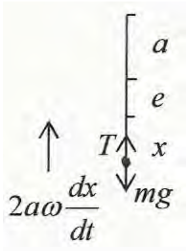
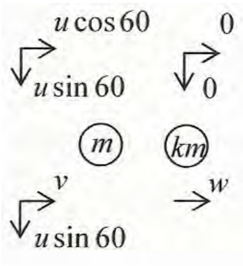
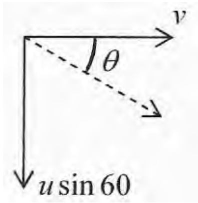
| Question Number | Scheme | Marks |
|---|---|---|
| 1. | (a) $\frac{1}{2}\frac{dv}{dt} = \frac{1}{2}g - 2v$ | M1 |
| | $\Rightarrow 5\frac{dv}{dt} = 49 - 20v$ (*) | A1 (2) |
| | (b) $\int \frac{5dv}{49-20v} = \int dt$ (separate variables) | M1 |
| | $\frac{-5}{20}\ln(49-20v) = t$ (+c) | A1 |
| | $t = 0, v = 0 \Rightarrow c = -\frac{1}{4}\ln 49$ (attempt to get c) | M1 |
| | $t = \frac{1}{4}\ln\left(\frac{49}{49-20v}\right)$ | |
| | $t = 1 : 1 = \frac{1}{4}\ln(\frac{49}{49-20v})$ (correct use of logs/exp) | M1 |
| | $\rightarrow v \approx 2.41ms^{-1}$ or $2.4ms^{-1}$ | A1 (5) |
| | | **Total 7 marks** |
| 2. | (a) Energy: $\frac{1}{2}m\left(\frac{37ga}{5} - v^2\right) = mg.2a(1-\cos\theta)$   | M1 A1 |
| | Using $\theta = \frac{\pi}{3}$ & solve: $\rightarrow v = \sqrt{\frac{27ga}{5}}$ (*) | M1 A1 (4) |
| | (b) Impact: $u_1 = ev\sin 30$   | M1 A1 |
| | KE loss $= \frac{1}{2}m(v^2\sin^2 30 - e^2v^2\sin^2 30)$ | |
| | $\left[+\frac{1}{2}mv^2\cos^2 30 - \frac{1}{2}mu_2^{\ 2}\right] = \frac{3mga}{5}$ | M1 A1 |
| | [Using $u_2 = v\cos 30$ if necessary & ] reducing to equation in (m, g, a) e alone | |
| | $\frac{3mga}{5} = \frac{1}{2}m.\frac{27ga}{5}.\frac{1}{4}(1-e^2)$ | A1 |
| | Solve for $e$: $\rightarrow e = \frac{1}{3}$ | M1 A1 (7) |
| | | **Total 11 marks** |

| Question Number | Scheme | Marks |
|---|---|---|
| 3. | (a) | |
| | (i) $\mathbf{v}_Q = {}_Q\mathbf{v}_P + \mathbf{v}_P$ | |
| | $\lvert\mathbf{v}_Q\rvert^2 = (10\cos 30)^2 + (16 - 10\sin 30)^2$ | M1 A1 |
| | $= 75 + 121$ | |
| | $\Rightarrow \underline{\lvert\mathbf{v}_Q\rvert = 14 ms^{-1}}$ | A1 |
| |  | |
| | (ii) $\tan\theta = \dfrac{16 - \sin 30}{10\cos 30}$ (o.e.) | M1 |
| | $\theta \approx 51.8°$, $\Rightarrow$ bearing $\underline{308°}$ (nearest degree) | A1, A1 |
| | | (6) |
| | (b) At nearest approach: $PN = 20\sin 30$ | M1 A1 |
| | $= \underline{10\ km}$ | A1 |
| |  | (3) |
| | (c) $Time = \dfrac{20\cos 30}{10} \approx 1.732\ hrs$ | M1 A1 |
| | $\Rightarrow \underline{Time \approx 4.44\ pm}$ (AWRT) | A1 |
| | | (3) |
| | | **Total 12 marks** |
| | <u>Alternatives</u> | |
| | (a) Use of cosine rule in velocity vector triangle. | |
| | (b) & (c) Use of scalar product of relative velocity and relative position or differentiating magnitude of relative position vector squared to find the minimum distance and time at which it occurs. | |

| Question Number | Scheme | Marks |
|---|---|---|
| 4. | (a) R(↓) $m\frac{d^2x}{dt^2} = mg - T - 2m\omega\frac{dx}{dt}$ ($\underline{\underline{4}}$ terms) | M1 A1 ↓ |
| | $m\frac{d^2x}{dt^2} = mg - \frac{2m\omega^2 a}{a}(e+x) - 2m\omega\frac{dx}{dt}$ | M1 ↓ |
| | $\rightarrow \underline{\frac{d^2x}{dx^2} + 2\omega\frac{dx}{dt} + 2\omega^2 x = 0}$ (*) | M1 A1 |
| |  | (5) |
| | (b) $x = e^{-\omega t}(A\cos\omega t + b\sin\omega t)$ | |
| | $t = 0, x = 0 \Rightarrow \underline{A = 0}$ | B1 |
| | $\frac{dx}{dt} = -\omega e^{-\omega t}.B\sin\omega t + e^{-\omega t}.B\omega\cos\omega t$ (use of product rule) | M1 |
| | $t - 0, \frac{dx}{dt} = U : U = B\omega \Rightarrow \underline{B = \frac{U}{\omega}}$ | M1 A1 |
| | | (4) |
| | (c) $\frac{dx}{dt} = -Ue^{-\omega t}\sin\omega t + Ue^{-\omega t}\cos\omega t = 0$ | M1 |
| | $\Rightarrow \tan\omega t = 1$ (solve for $\tan\omega t$) | M1 |
| | $\Rightarrow \underline{t = \frac{\pi}{4\omega}}$ | A1 |
| | | (3) |
| | | **Total 12 marks** |

| Question Number | Scheme | Marks |
|---|---|---|
| 5. | (a) $CLM(\leftrightarrow): mu\cos 60 = mv + kmw$ | M1 A1 |
| | $NLI: \frac{1}{2}u\cos 60 = w - v$ | M1 A1 |
| |  | |
| | Solve for w: $(1+k)w = \frac{1}{2}u\left(1+\frac{1}{2}\right)$ | M1 |
| | $\Rightarrow w = \frac{3u}{4(k+1)}$ (*) | A1 (6) |
| | (b) Solve for v $\rightarrow v = \frac{u(2-k)}{4(k+1)}$ | M1 A1 |
| | $\tan\theta = 2\sqrt{3} = \frac{u\sin 60}{v}$ | M1 A1 |
| | $= \frac{u\sqrt{3}}{2}.\frac{4(k+1)}{u(2-k)}$ | |
| |  | |
| | Solve k: $\rightarrow k = \frac{1}{2}$ | M1 A1 (6) |
| | (c) $k = \frac{1}{2} \Rightarrow v = \frac{u}{4}, w = \frac{u}{2}$ | B1 |
| | KE loss $= \frac{1}{2}mu^2 - \left(\frac{1}{2}m.\frac{u^2}{16} + \frac{1}{2}m.\frac{3u^2}{4} + \frac{1}{2}.\frac{1}{2}m.\frac{u^2}{4}\right)$ | M1 A1 |
| | $= \frac{1}{2}mu^2\left(1 - \frac{1}{16} - \frac{3}{4} - \frac{1}{8}\right)$ | |
| | $= \frac{1}{32}mu^2$ | A1 (4) |
| | | **Total 16 marks** |

| Question Number | Scheme | Marks |
| --- | --- | --- |
| 6. | (a) PE of R $= -\sqrt{2}mga\cos 2\theta$ (+c) (1) | B1 |
| | PE of LH mass $= -\frac{3}{2}mg\left(2a - 2a\sin(45+\theta)\right)$ (+c) (2) | M1 A1 |
| | PE of RH mass $= -\frac{3}{2}mg\left(2a - 2a\sin(45-\theta)\right)$ (+c) (3) | A1 |
| | $V$ = (1) + (2) + (3) (in terms of $\theta$ etc.) | M1 |
| | $= -\sqrt{2}mga\cos 2\theta - \frac{3}{2}mg\left[4a - a\sqrt{2}(\cos\theta + \sin\theta + \cos\theta - \sin\theta)\right]$ | M1 |
| | $= -\sqrt{2}mga\cos 2\theta - \frac{3}{2}mga\left(-2\sqrt{2}\cos\theta + 4\right)$ | |
| | $= \sqrt{2}mga(3\cos\theta - \cos 2\theta)$ + constant (*) | A1 (7) |
| | (b) $\frac{dV}{d\theta} = \sqrt{2}mga(-3\sin\theta + 2\sin 2\theta)$ | M1 A1 |
| | $\frac{dV}{d\theta} = 0 \Rightarrow 2\sin 2\theta - 3\sin\theta = 0$ | M1 |
| | $\Rightarrow \sin\theta(4\cos\theta - 3) = 0$ | M1 |
| | $\Rightarrow \theta = 0,$ or $\theta = \pm\arccos\frac{3}{4}(= \pm 0.723)$ | A1, A1 (6) |
| | (c) $\frac{d^2V}{d\theta^2} = \sqrt{2}mga(-3\cos\theta + 4\cos 2\theta)$ | M1 A1 |
| | $\cos\theta = \frac{3}{4}: \frac{d^2V}{d\theta^2} = \sqrt{2}mga\left(-3.\frac{3}{4} + 4\left(2.\frac{9}{16} - 1\right)\right)$ | M1 |
| | $= \sqrt{2}mga\left(-\frac{9}{4} + \frac{1}{2}\right)$ | |
| | $<0$ $\therefore$ Unstable | A1 (4) |
| | | **Total 17 marks** |